

# Evolution of isolated neutron stars till accretion. The role of initial magnetic field

P.A. Boldin<sup>1</sup>, S.B. Popov<sup>2</sup> \*

<sup>1</sup>*National Research Nuclear University "MEPhI", Kashirskoe shosse 31, Moscow, 115409, Russia*

<sup>2</sup>*Sternberg Astronomical Institute, Universitetski pr. 13, Moscow, 119991, Russia*

Accepted ..... Received .....; in original form .....

## ABSTRACT

We study evolution of isolated neutron stars on long time scale and calculate distribution of these sources in the main evolutionary stages: Ejector, Propeller, Accretor, and Georotator. We compare different initial magnetic field distributions taking into account a possibility of magnetic field decay, and include in our calculations the stage of subsonic Propeller.

It is shown that though the subsonic propeller stage can be relatively long, initially highly magnetized neutron stars ( $B_0 \gtrsim 10^{13}$  G) reach the accretion regime within the Galactic lifetime if their kick velocities are not too large. The fact that in previous studies made >10 years ago, such objects were not considered results in a slight increase of the Accretor fraction in comparison with earlier conclusions. Most of the neutron stars similar to the Magnificent seven are expected to become accreting from the interstellar medium after few billion years of their evolution. They are the main predecessors of accreting isolated neutron stars.

**Key words:** stars: neutron — pulsars: general

## 1 INTRODUCTION

Accreting isolated neutron stars (AINS) were predicted 40 years ago by Shvartsman (1971) and independently by Ostriker et al. (1970). In early 90s there was some enthusiasm due to the launch of the ROSAT satellite, which was expected to find many sources of this kind (Treves & Colpi 1991). Several population studies have been made (Blaes et al. 1990; Blaes & Rajagopal 1991; Blaes & Madau 1993; Madau & Blaes 1994; Blaes et al. 1995; Manning et al. 1996). However, it came out that AINS, if they exist, are very elusive (Colpi

et al. 1998). The main reason is that initial (kick) velocities of NSs appeared to be significantly larger, than it have been thought before (Lyne & Lorimer 1994). Initial guess that the number of Accretors is small due to low luminosity of high-velocity NSs was shown to be wrong. In a detailed study by Popov et al. (2000) (hereafter Paper I) it was shown that INS with Crab-like initial parameters and constant magnetic fields spend all their lives as Ejectors (we follow the classification summarized in Lipunov 1992) if their initial velocities are  $\gtrsim 100$  km s<sup>-1</sup>. Then, the fraction of Accretors was mainly determined by the fraction of low-velocity NSs.

Up to the very end of 90s, it was believed that the vast majority of NSs are born sim-

\* E-mail: polar@sai.msu.ru (SBP)

ilar to the Crab pulsar. I.e., that they have short initial spin periods (from milliseconds to few tens of millisecond) and magnetic fields  $B \sim 10^{12}$  G. Now it is believed, that about one half of NSs have different initial properties (Popov et al. 2006; Keane & Kramer 2008). There are at least three groups of sources with distinct parameters: compact central objects (CCOs) in supernova remnants (SNR), magnetars (anomalous X-ray pulsars - AXPs, and soft gamma-ray repeaters - SGRs), and cooling radio-quiet NSs dubbed the Magnificent seven (M7) (Popov 2008). CCOs have low initial fields  $\sim 10^{11}$  G (Halpern et al. 2007; Gotthelf & Halpern 2009) and relatively long spin periods (hundreds of millisecond). AXPs and SGRs have large fields  $\sim 10^{14}$  G (see a review in Mereghetti 2008). The Magnificent seven-like NSs have fields slightly above  $10^{13}$  G (Haberl 2007; Kaplan 2008). Probably, some of rotating radio transients (RRATs, McLaughlin et al. 2006) are similar to the M7. This variety in initial properties deserves new studies of evolving NSs using the population synthesis technique (see a review in Popov & Prokhorov 2007). In this paper we present the first step.

We describe two models. At first, we discuss a simple semianalytical approach, which is used to illustrate the main features of the scenario. In this model velocities and ambient densities are not changing. Then we present a detailed numerical model, which takes into account spatial movements of NSs in the Galactic potential and realistic 3D distribution of the interstellar medium (ISM). Our main results are based on this model.

In the next section we present basic concepts used in both models, and describe each of them. Then, in Sec. 3, we present results. Discussion is given in Sec.4. In the last section we present our conclusions.

## 2 MODELS

In this section we describe our models. We start with explanation of some basic processes and parameters of magneto-rotational evolution used in both models. Then we discuss the semianalytical and the full numerical model, consequently.

### 2.1 Basic processes and parameters

Here we describe some aspects of magneto-rotational evolution implemented in both semi-analytical and numerical models.

#### 2.1.1 Standard magneto-rotational evolution

Here we mainly follow the approach described in Lipunov (1992). We consider a NS being born as an Ejector. At this stage a relativistic wind and Poynting flux are so strong that they prevent incoming matter to penetrate inside neither gravitational capture radius,  $R_G$ , nor inside the light cylinder radius,  $R_l$ .  $R_G$  represent the typical scale at which the ISM is captured by the NS gravity:

$$R_G = 2GM/v_{\text{rel}}^2, \quad (1)$$

where  $M$  is a NS mass and  $v_{\text{rel}}$  is a relative velocity of the NS and the ISM. The light cylinder radius is defined as:

$$R_l = c/\omega, \quad (2)$$

where  $\omega$  is the spin frequency of a NS. The ejected matter creates a cavern in the ISM within the distance of the Shvartsman radius,  $R_{\text{sh}}$ , at which the magneto-dipole pressure,  $P \sim \mu^2/R_l^4 R_{\text{sh}}^2$ , is equal to the ram pressure of the ISM,  $P \sim \rho v_{\text{rel}}^2$ . At this stage a young NS can be visible as a radiopulsar (PSR), and we assume that it losses energy via relativistic wind and Poynting flux according to the magneto-dipole formula:

$$\frac{1}{2} \frac{dI\omega^2}{dt} = -\frac{2}{3} \frac{\mu^2 \omega^4}{c^3} \sin^2 \chi. \quad (3)$$

Here  $I = 10^{45}$  g cm<sup>2</sup> is a moment of inertia of a NS,  $\mu$  is a magnetic dipole moment,  $\chi$  is an angle between rotational and magnetic axis, which is assumed to be  $\pi/2$  everywhere below.

The Ejector stage finish when the Shvartsman radius,  $R_{\text{sh}}$ , becomes less than  $R_G$ . The regime changes because after matter appear inside  $R_G$ , its pressure start to grow  $P_{\text{matter}} \sim r^{-5/2}$ . This more rapidly than the relativistic wind pressure growth:  $P_{\text{wind}} \sim r^{-2}$ . So, the condition  $P_{\text{matter}} > P_{\text{wind}}$  is reached and the pulsar wind cannot stop the incoming flow.

Another reason that causes the Ejector stage to cease is disappearance of the magneto-dipole emission. This happens when  $R_{\text{sh}}$  becomes less than the light cylinder radius. So, now matter fills the light cylinder, preventing the creation of the magneto-dipole emission.

For isolated NSs both cases can happen not only due to spin-down, but also because an object enters a more dense region of the ISM. Variations of the relative velocity of a NS and the ISM are important, too. For small velocities  $R_G$  can become larger than  $R_{sh}$ . Oppositely, for large velocities  $R_{sh}$  can become smaller than the light cylinder radius due to the ram pressure (in this case  $R_G < R_l$  due to a large velocity).

To summarize, when at least one of conditions  $R_{sh} < R_G$  or  $R_{sh} < R_l$  is met, the Ejector stage ceases. After that, matter falls down till its pressure is counterbalanced by the magnetic field pressure. The radius at which these pressures are equalized is called the Alfvén radius:

$$R_A = \left( \frac{\mu^2}{2\dot{M}\sqrt{2GM}} \right)^{2/7} \quad (4)$$

for the case  $R_A < R_G$  and

$$R_A = \left( \frac{2\mu^2 G^2 M^2}{\dot{M} v_{rel}^5} \right)^{1/6} \quad (5)$$

for the opposite.  $\dot{M}$  is the accretion rate.

The Propeller stage begins if  $R_A > R_c$ , where

$$R_c = \left( \frac{GM}{\omega^2} \right)^{1/3} \quad (6)$$

is the corotation radius at which the solid body rotational velocity equals the escape velocity. At this stage matter is “propelled” away from a star, because of its interaction with the magnetosphere.

For the propeller stage we use the model proposed by Shakura (1975). The same approach was also used in Paper I. The period derivative can be written as:

$$\frac{dP}{dt} = \dot{M} R_A^2 P I^{-1} \simeq K P^\alpha \text{ s s}^{-1}. \quad (7)$$

For  $\alpha = 1$  we obtain:

$$K = 2.4 \times 10^{-14} \mu_{30}^{8/7} n^{3/7} v_{10}^{-9/7} \text{ s}^{-1}, \quad (8)$$

where  $\mu_{30}$  is a magnetic dipole moment in units of  $10^{30} \text{ G cm}^{-3}$ ,  $n$  is the ISM number density,  $v_{10}$  is the total velocity  $\sqrt{a_s^2 + v_{rel}^2}$  in units of  $10 \text{ km s}^{-1}$ ,  $a_s = 10 \text{ km s}^{-1}$  is the sound speed.

When a NS spin-downs enough for the condition  $R_c > R_A$  to be met, a star leaves the Propeller stage and switches to the next stage, depending on the relation between  $R_A$  and  $R_G$ . Note, that in our scenario this can also happen because of changes in the ISM density or in the velocity of a NS.

If  $R_A > R_G$ , then a NS enters the Georotator stage, which is called so because of similarity of the NS magnetosphere structure to the Earth magnetosphere in the fast solar wind. At this stage in our model no spin-up/spin-down mechanisms are taken into account.

Otherwise, if  $R_A < R_G$  then a star at first enters the subsonic Propeller stage, where the main accretion mechanism – the Rayleigh-Taylor instability – is suppressed by high temperature of an envelope (i.e., the gas is “too light”). Temperature increases because of heating (a NS loses angular momentum to the envelope and heats it), and decreases because of radiative losses (mainly bremsstrahlung). The spin-down rate at this stage is taken in the following form (Davies & Pringle 1981):

$$\frac{dP}{dt} = 2.4 \times 10^{-11} \mu_{30}^2 m^{-1} \text{ s s}^{-1}. \quad (9)$$

Note, that  $\dot{P}$  is constant if the field is not decaying.

A NS starts to accrete from the ISM and enters the Accretor stage when heating of an envelope due to rotational energy losses by a NS becomes less effective than bremsstrahlung cooling, and so the matter at the magnetospheric boundary becomes so heavy that the Rayleigh-Taylor instability develops. This occurs at the critical period (Ikhsanov 2001):

$$P_{break} = 8.7 \times 10^4 R_{A,10}^{5/2} \mu_{30}^{-2/3} m^{1/6} \text{ s} \quad (10)$$

where  $R_{A,10}$  is the Alfvén radius in units of  $10^{10} \text{ cm}$ . At the Accretor stage the spin-down is calculated as at the subsonic Propeller stage (eq. 9), and we neglect possible quasi equilibrium at this stage. This is an oversimplification (Prokhorov et al. 2002), but as here we are not interested in details of spin properties of Accretors we neglect some details.

### 2.1.2 Magnetic field distribution and decay. Initial period distribution

We consider two different field distributions. The first is a delta-function. As a standard value we use  $\mu_{30} = 1$ , which gives us  $B_{eq} = 10^{12} \text{ G}$ , according to  $\mu = B_{eq} R^3$ . This distribution is chosen to make comparison with the results from Paper I.

The second distribution is a result of the magnetic field decay starting with the “optimal” one from the paper Popov et al. (2010). This “optimal” distribution is the lognormal one with

$\langle \log(B_{\text{pole}}/[G]) \rangle = 13.25$  and  $\sigma_{\log B_{\text{pole}}} = 0.6$ , where  $B_{\text{pole}}$  is the value of the poloidal field on the magnetic pole ( $B_{\text{pole}} = 2B_{\text{eq}}$ ). Till the magnetic field reaches some saturation value  $B_{\text{min}}$  it undergoes decay according to

$$B(t) = B_0 \frac{e^{-t/\tau_{\text{Ohm}}}}{1 + \frac{\tau_{\text{Ohm}}}{\tau_{\text{Hall}}} (1 - e^{-t/\tau_{\text{Ohm}}})} \quad (11)$$

where  $\tau_{\text{Ohm}}$  is the Ohmic characteristic time, and  $\tau_{\text{Hall}}$  is the typical timescale of the fast, initial Hall stage, that depends on the initial field ( $\tau_{\text{Hall}} \propto 1/B_0$ ). Typically,  $\tau_{\text{Ohm}} = 10^6$  yrs and  $\tau_{\text{Hall}} = 10^4$  yrs (for  $B_0 = 10^{15}$  G). The asymptotic value of the field depends of the initial strength. In order to approximate the results of simulations by Pons et al. (2009) we choose

$$B_{\text{min}} = \min \left\{ \frac{B_0}{2}, 2 \times 10^{13} \text{ G} \right\} \quad (12)$$

Since, according to eq. (16)  $\min(t_E) \approx 10^7$  yr  $\gg \tau_{\text{Ohm}} = 10^6$  yr, we neglect the process of decay and take the field already decayed down to  $B_{\text{min}}$  as the initial field in our model. The shape of this distribution is shown in Fig. 3.

Below we will refer to these two distributions as “the standard” (for  $\mu_{30} = 1$ ) and “the decayed” respectively. Everywhere below  $B$  is the polar field.

For  $P_0$  in the complete model we take the distribution with  $\langle P_0 \rangle = 0.25$  s and  $\sigma_{P_0} = 0.1$  s. Such distribution was used in Popov et al. (2010). In our scenario here this is a simplification, as a NS can spin-down while its field decays down to the minimum value. So, an initial period in our model should be different (longer, depending on the strength of the initial magnetic field) from a period used in Popov et al. (2010). This results in slight overestimating of the number of objects at the Ejector stage on the price of other stages. But we tested that this assumption does not influence our results significantly even for magnetar-scale initial fields. This is so because a star always evolves off the Ejector stage much slower than the field decays.

### 2.1.3 Velocity distribution

A NS initial velocity (and so, its kick) has great impact on its magneto-rotational evolution. This is due to the fact that the efficient accretion rate  $\dot{M}$  strongly depends on the velocity:  $\dot{M} \sim v_{\text{rel}}^{-3}$ . Note, that  $\dot{M}$  (i.e. just a combination of the ISM density, relative velocity and NS mass) can be

defined for any evolutionary stage, and it just demonstrates how efficiently a NS interacts with the surrounding medium. Almost all characteristic radii and critical periods depend on  $\dot{M}$ .

Not only the absolute value of a kick is important, its direction is significant, too. If a kick has large component perpendicular to the Galactic disc,  $v_z$ , then a star spends much less time close to the Galactic plane, where the ISM density is higher (see below).

Most of Accretors in our scenario have small velocities, so only low-velocity end of the distribution is important. Several shapes of the kick velocity distribution have been discussed in the literature: Arzoumanian et al. (2002); Hobbs et al. (2005); Faucher-Giguère & Kaspi (2006). Here we use the initial velocity distribution proposed by Arzoumanian et al. (2002). It is a bimaxwellian distribution with the Gaussian three-dimensional dispersions  $\sigma_1 = 90$  km s $^{-1}$  and  $\sigma_2 = 500$  km s $^{-1}$ . We vary the contribution of each of these components using the parameter  $w_1$ , which is the fraction of NSs in the low-velocity component.

## 2.2 Semianalytical model

In a simple semianalytical model we assume monotonic magneto-rotational evolution of a NS in constant conditions: NS velocity, its magnetic field and the ISM density do not change during a NS lifetime. Each transition from stage to stage is defined by solving appropriate equations for the given characteristic values. In this model we always use the NS mass  $M = 1.4M_{\odot}$  ( $m = 1.4$ ) and the initial period  $P_0 = 0.02$  s.

At first, we solve equations  $R_{\text{sh}}(P, \mu, v) = R_G$ ,  $R_{\text{sh}}(P, \mu, v) = R_1(P)$  to find the period at which the Ejector stage ends and the star becomes a Propeller. This gives us:

$$P(E \xrightarrow{G} P) \approx 7 \mu_{30}^{1/2} v_{10}^{1/2} n^{-1/4} m^{-1/2} \text{ s} \quad (13)$$

for  $R_{\text{sh}} = R_G$  and

$$P(E \xrightarrow{LC} P) \approx 142 \mu_{30}^{1/3} v_{10}^{-1/3} n^{-1/6} \text{ s} \quad (14)$$

for  $R_{\text{sh}} = R_1$ . The critical period (for any branch of the transition from the Ejector stage) is called below  $P_E$

Integrating eq. (3) we obtain:

$$P(t) = \sqrt{P_0^2 + \frac{16\pi^2 \mu^2}{3c^3 I} t} \text{ s.} \quad (15)$$

Then, neglecting  $P_0$  we derive the time of the first transition:

$$t(E \xrightarrow{G} P) \approx 8.25 \times 10^8 \mu_{30}^{-1} v_{10} n^{-1/2} m^{-1} \text{ yrs}, (16)$$

$$t(E \xrightarrow{LC} P) \approx 3.26 \times 10^{11} \mu_{30}^{-4/3} v_{10}^{-2/3} n^{-1/3} \text{ yrs}. (17)$$

Among these values the smaller one is used.

If for a given  $\mu$  and  $v$  the value of  $t_E$  exceeds  $t_{Gal} = 10^{10}$  yr, then the fraction of lifetime which a star spends as an Ejector,  $\tau_E$ , is equal to 1. Elsewhere,  $\tau_E$  is equal to  $t_E/t_{Gal}$ .

Next, we must consider the Propeller stage. Solving the equation  $R_c(P) = R_A(\mu, v)$  for both  $-R_A > R_G$  and  $R_A < R_G$  – cases, we obtain the critical periods:

$$P(P \xrightarrow{A < G} ssP) = 500 \mu_{30}^{6/7} v_{10}^{9/7} n^{-3/7} m^{-11/7} \text{ s}, (18)$$

$$P(P \xrightarrow{A > G} G) = 3 \times 10^5 \mu_{30}^{1/2} v_{10}^{-1/2} n^{-1/4} m^{-1/2} \text{ s}. (19)$$

These periods correspond to the end of the Propeller stage. Any of such periods is called below  $P_P$  (note, that before in several papers such a period was called  $P_A$ , as without the subsonic Propeller stage and neglecting the possibility that a star becomes a Georotator, after reaching  $P_P$  accretion starts).

Then by solving the equation  $\log(P_A) - \log(P_E) = K\Delta t$  on  $\Delta t$  we obtain<sup>1</sup> the value for  $\Delta t_P$ . This is the time period during which a NS stays at the Propeller stage. The fraction of Propellers in the total distribution among evolutionary stages is taken in the form  $\tau_P = \min(1 - \tau_E, \Delta t_P/t_{Gal})$ .

If  $R_G(v) < R_A(\mu, v) \leq R_c(P_P)$  a star is considered as a Georotator. The fraction of NSs at this stage is  $\tau_G = 1 - (\tau_E + \tau_P)$ .

If  $R_A(\mu, v) < R_G(v)$ , and due to the spin-down at the Propeller stage  $R_A(\mu, v) \leq R_c(P_A)$ , then the subsonic Propeller stage begins. The spin-down during this stage is given by eq. (9). The stage ends when  $P = P_{break}$ , which gives us the equation:  $\Delta t = (P_{break} - P_A)/\dot{P}$ , where  $\dot{P}$  is taken according to eq. (9), from which we obtain  $\Delta t_{ssP}$  – the duration of the subsonic Propeller stage. Fraction of this stage is  $\tau_{ssP} = \min(1 - (\tau_E + \tau_P), \Delta t_{ssP}/t_{Gal})$ .

After a NS spin-downs and leaves the subsonic Propeller stage, it starts to accrete. The fraction of Accretors is  $\tau_A = 1 - (\tau_E + \tau_P + \tau_{ssP})$ .

<sup>1</sup>  $P_E$  is taken as  $P(E \xrightarrow{G} P)$  for the case  $R_A < R_G$ , and  $P(E \xrightarrow{LC} P)$  – for the opposite case.

For the scenario without the subsonic Propeller stage we assumed  $\tau_{ssP} = 0$  (or, equivalently, taking  $\tau_A = 1 - (\tau_E + \tau_P)$ , as it was done in Paper I).

## 2.3 Complete numerical model

Using the same subroutines for characteristic values and spin-down rates as in the semianalytical model, we make a more detailed numerical model. A NS evolution in this model proceeds in realistic conditions: the Galactic potential and the ISM density distribution.

This is done by splitting all the time from 0 to  $t_{Gal}$  into an equidistant grid. Then, on the domain of the acquired time grid we compute all values which do not depend on the rotation of a star: velocities, coordinates, ISM densities,  $R_A$ ,  $\dot{M}$ , and so on.

After all rotation-independent values are fetched, the magneto-rotational evolution is calculated starting with the Ejector stage. Transitions between stages are determined by changes in relations between characteristic values, as described above. Note, that there is almost no prohibited transitions, because of large variations in the environment and in the velocity of a NS.

### 2.3.1 Spatial evolution

Magneto-rotational evolution strongly depends on the spatial evolution of a NS, moving through the Galaxy. A NS receives an initial kick velocity. The vectors of the kick and the progenitor's Keplerian velocities are summed. In our model, the ISM rotates with a Keplerian velocity around the center of the Galaxy.

Equations of motion of a NS with a given initial values are solved on the time grid using the LSODA subroutine (Hindmarsh 1983). The Galactic potential is taken in the same form as in Paper I (i.e. in the form suggested in Miyamoto & Nagai 1975 and Paczyński 1990). It is a three component potential (disc, bulge, and halo) which reproduces well enough trajectories on a long time scale (billions of years). In the problem considered here we do not need a more complicated gravitational potential.

### 2.3.2 ISM density and NS initial spatial distribution

The ISM density is taken according to the old analytical model in Posselt et al. (2008). It is generally the same as in Paper I with some corrections in the  $z$ -dependence (see below). This distribution have exponential or Gaussian behaviour perpendicular to the Galactic disc. The radial distribution has a peak at  $R \sim 5$  kpc. For very low-density regions (large  $R$  and  $z$ ) we used the minimum value of the ISM number density  $n = 10^{-5} \text{ cm}^{-3}$ .

In Paper I there was a small mistake (copied from Zane et al. 1995) regarding the ISM distribution. Dispersions in eqs. (5) and (7) of Paper I should be divided by 2.35 (as they are actually not dispersions, but FWHM). In eq. (6) of Paper I the coefficients 0.345, 0.107, and 0.064 should be 0.7, 0.19, and 0.11, correspondently.

Exactly as in Paper I, the birthrate of NSs is proportional to the square of the local ISM density.

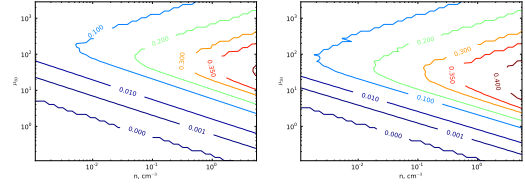
## 3 RESULTS

### 3.1 Semianalytical model results

With this model we address two main questions. How does the account for the subsonic Propeller stage influence the fraction of Accretors? How does the fraction of accretors depend on the magnetic field?

In Paper I the subsonic Propeller stage was not used, and only Crab-like fields,  $\sim 10^{12}$  G, were considered. Here we include this stage and take into account higher magnetic fields (up to the values typical for decayed fields of magnetars). Obviously, the first effect reduces the number of Accretors, while the second – increases. It is interesting to understand with a simple model the interplay between them before addressing the same questions with a more advanced one. In this model we use only the delta-function magnetic field distribution.

In Fig. 1 we show the dependence of the fraction of INs at the Accretor stage on the magnetic field and the ISM number density. Here both of these parameters are constant during a NS lifetime. For Crab-like fields and the ISM density  $\sim 0.1 - 1 \text{ cm}^{-3}$  it is below  $\sim 1\%$ . The number of Accretors for a given realistic ISM density steadily grows with increasing initial magnetic fields up to  $\sim 5 \times 10^{13}$  G. This is related to



**Figure 1.** Fraction of Accretors in the semianalytical model. The bi-maxwellian kick velocity distribution from Arzoumanian, Chernoff and Cordes (2002) is taken. In the left panel we show results for the scenario with the subsonic Propeller stage. In the right – we neglect this stage to demonstrate its influence on our results.

the fact that a NS with larger magnetic field spin-downs faster, and so quicker reaches the stage of accretion. However, for larger fields it happens at large periods, but the increase of the spin-down rate is more important than the increase of critical periods for transitions.

For large fields the fraction of Accretors starts to decrease, because more and more sources appear as Georotators due to their huge magnetospheres. However, it is important to repeat, that here we assume constant fields, but it is normally accepted that large fields gradually decay while a NS is aging.

The difference with Paper I, where for  $\mu \approx 10^{30} \text{ G cm}^3$ ,  $n \approx 1 \text{ cm}^{-3}$  and comparable velocities we obtained  $\sim$  several percents of Accretors, is explained by the influence of the subsonic Propeller stage. It is visible in the right panel. The fraction of accretors for  $n = 1 \text{ cm}^{-3}$  and  $\mu = 10^{30} \text{ G cm}^3$  is about one percent if the subsonic Propeller stage is not taken into account. For the initial field in the M7 range the fraction of Accretors in our new model (with the subsonic Propeller stage included, see the left panel) goes up to  $\sim 10 - 30\%$ .

Clearly, the main conclusion here is that the subsonic Propeller is not a strong barrier for NSs with realistically large fields in contrast with conclusion by Ikhsanov (2001). As now we know that the fraction of such objects (SGRs, AXPs, M7, RRATs) is not low – tens of percent, – one can expect that significant fraction of old INs can start to accrete. Thus, one has to study the distribution of INs in different evolutionary stages in more details.

Some results of this subsection are summarized in the Table. 1. We demonstrate twelve tracks for different  $\mu, n, v$ . NSs following tracks II, III, and IX always stay at the Ejector stage. High

**Table 1.** NS evolutionary tracks

Track	$n, \text{cm}^{-3}$	$\mu_{30}$	$v_{10}$	$\tau_E$	$P_E, \text{s}$	$\tau_P$	$P_P, \text{s}$	$\tau_{ssP}$	$P_{\text{break}}, \text{s}$
Track I	0.5	1	5	0.419	16.051	0.423	$3.163 \times 10^3$	0.850	$2.278 \times 10^6$
Track II	0.5	1	20	—	—	—	—	—	—
Track III	0.5	1	40	—	—	—	—	—	—
Track IV	0.5	10	5	0.042	50.758	0.042	$2.276 \times 10^4$	0.067	$1.317 \times 10^7$
Track V	0.5	10	20	0.168	101.517	0.170	$1.353 \times 10^5$	0.651	$2.568 \times 10^8$
Track VI	0.5	10	40	0.163	100.091	0.169	$1.523 \times 10^5$	Georotator	
Track VII	2.0	1	5	0.209	11.350	0.212	$1.746 \times 10^3$	0.370	$8.464 \times 10^5$
Track VIII	2.0	1	20	0.838	22.700	0.854	$1.038 \times 10^4$	—	—
Track IX	2.0	1	40	—	—	—	—	—	—
Track X	2.0	10	5	0.021	35.892	0.021	$1.257 \times 10^4$	0.030	$4.892 \times 10^6$
Track XI	2.0	10	20	0.084	71.783	0.085	$7.469 \times 10^4$	0.264	$9.541 \times 10^7$
Track XII	2.0	10	40	0.103	79.442	0.106	$1.077 \times 10^5$	Georotator	

velocity NSs with fields larger than the Crab-like value, become Georotators after the Propeller stage (tracks VI and XII). The NS following track VIII never becomes an Accretor as it stays for a long time at the subsonic Propeller stage. All the rest NSs (tracks I, IV, V, VII, X, XI) finally start to accrete.

### 3.2 Complete numerical model

In this subsection we present our main results obtained with the complete model for  $10^5$  evolutionary tracks. Our main aim is to calculate the distribution in evolutionary stages for two different distributions of the initial magnetic field. In addition, we demonstrate the effect of changing velocity distribution.

As we use the bi-Maxwellian distribution proposed by Arzoumanian et al. (2002), to demonstrate the dependence of our results on the velocity distribution, we decided to change relative contributions of the two constituents. In Fig.2 the horizontal axis shows  $w_1$  – the contribution of the low-velocity part of the bi-Maxwellian distribution. For  $w_1 = 0$  we have a pure Maxwellian distribution with  $\sigma = 500 \text{ km s}^{-1}$ , for  $w_1 = 1$  – a pure Maxwellian with  $\sigma = 90 \text{ km s}^{-1}$ .

We show results of calculations for two distributions of initial magnetic fields described above (sec.2.1.2). The first is just a delta-function  $\mu = 10^{30} \text{ G cm}^3$ . It corresponds to the typical assumption made in 90s. The second is based on recent results by Popov et al. (2010).

Fractions of Ejectors, Propellers, subsonic Propellers, Accretors and Georotators demonstrate monotonic, nearly linear behavior. The number of Ejectors strongly decreases with in-

creasing  $w_1$ . The behavior of Accretors and subsonic Propellers is opposite.

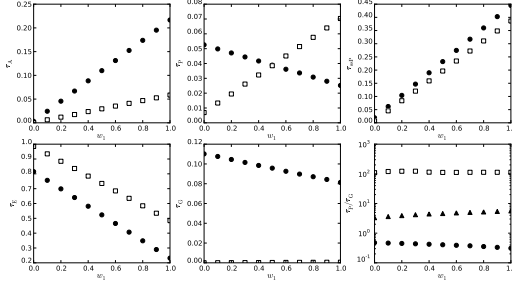
The behavior for the two studied field distributions is similar in the cases of Accretors, Ejectors, and subsonic Propellers. In the case of Propellers and Georotators the situation is different for two distributions.

In the most realistic case according to Arzoumanian et al. (2002), –  $w_1 = 0.4$  – we have (in the case of initially decayed field distribution)  $\sim 55\%$  of Ejectors,  $\sim 5\%$  of supersonic and  $\sim 20\%$  of subsonic Propellers,  $\sim 10\%$  of Accretors, and finally,  $\sim 10\%$  of Georotators. As we see, now for our “the best choice” model we predict more Accretors than in Paper I. It is what was expected on the basis of the semianalytical model.

The increase in the relative number of Accretors is due to the presence of INs with large initial magnetic fields. This is illustrated in Fig.3. We show there contributions of INs with different initial magnetic fields to the population of Accretors. Note, that the scale is logarithmic in both axis. INs with initial fields  $< 3 \cdot 10^{12} \text{ G}$  are more numerous than those with  $10^{13} \text{ G} < B < 2 \cdot 10^{13} \text{ G}$ .<sup>2</sup> However, the latter produce seven times more Accretors. Still, many (about 1/2) of NSs with the largest initial field considered here do not produce many Accretors as they become Georotators due to large spatial velocities.

Typically, NSs become Accretors in the regions of the Galactic disc, where the ISM density is higher. As it was noted before NSs with

<sup>2</sup> Here we speak about decayed fields in our model. If we recalculate it to obtain real pre-decay initial fields, then the intervals are changed.



**Figure 2.** Different panels show fractions of INs at different stages in the complete numerical model: Accretors (top left), Propellers (top center), subsonic Propellers (top right), Ejectors (bottom left) and Georotators (bottom center). Black circles denote the *decayed* magnetic field distribution, whereas empty squares corresponds to the delta-function  $\mu = 10^{30} \text{ G cm}^3$ . Bottom right panel shows the ratio of Propellers to Georotators. Triangles correspond to an additional variant of magnetic field distribution. In this variant only low-field –  $B \leq 5 \times 10^{12} \text{ G}$  – INs are taken from the *decayed* model. By  $w_1$  we mark the fraction of low-velocity component of the bi-maxwellian distribution from Arzoumanian, Chernoff and Cordes (2002). Here  $\mu = BR^3/2$ , as  $B$  is the magnetic field value at the pole.

low total velocity but significant  $z$ -component,  $v_z$ , spend most of its lifetime outside the Galactic plane. Most likely the longest stage for such a NS is the subsonic Propeller. If a star is born relatively far from the Galactic center and receives a large kick then it escapes from the Galaxy. Such a NS spends most of its life as an Ejector. Alternatively, if the velocity is high but not enough to escape the Galaxy, the NS returns to the Galactic plane after some long time, then it can quickly pass the Propeller stage (so-called *non-gravitating Propeller* in this case) and become a Georotator.

In addition to the global distribution we compute separately distributions over stages inside ( $R < 16 \text{ kpc}$  and  $|z| < 1 \text{ kpc}$ ) and outside the Galaxy (in the following paragraphs we refer as “the Galaxy” only to the former volume). It can seem surprising and confusing, but according to our model we predict more Accretors than Ejectors inside the Galaxy:  $\tau_A \sim 30\%$  and  $\tau_E \approx 20\%$ . Subsonic Propellers are more abundant than Accretors and Ejectors in this volume:  $\tau_{\text{ssP}} \sim 43\%$ . These numbers can be explained in the following way. Most of NSs which contribute a lot to the number of Accretors have  $v_z < 100 \text{ km s}^{-1}$ . They spend most of their lives inside the Galaxy, and so there they dominate. Other stages, corre-

spondently, are not abundant inside the Galaxy: Georotators contribute  $\tau_G \sim 7\%$ , Propellers –  $\tau_P \sim 1 - 2\%$ . Roughly, NSs with kick velocities from the low-velocity part of the distribution (about 30-40 %) stay inside the Galaxy. Those with magnetic fields higher than typical radio pulsar values become Accretors.

The situation outside the cylinder  $R < 16 \text{ kpc}$  and  $|z| < 1 \text{ kpc}$  is the following. Ejectors contribute  $\tau_E = 76\%$ , Georotators –  $\tau_G \approx 11\%$ , subsonic Propellers –  $\tau_{\text{ssP}} = 9\%$ , Propellers –  $\tau_P \approx 4\%$ , and Accretors –  $\tau_A \approx 0.2\%$ . As one can see, the situation with Ejectors and Accretors is opposite in comparison with the internal part. Almost all Accretors are situated inside the Galaxy. Note, that due to this Fig. 3 with the total distribution also refers to the population of Accretors inside the Galaxy. Almost 2/3 of all neutron stars are outside the Galaxy and they are either Ejectors or Georotators.

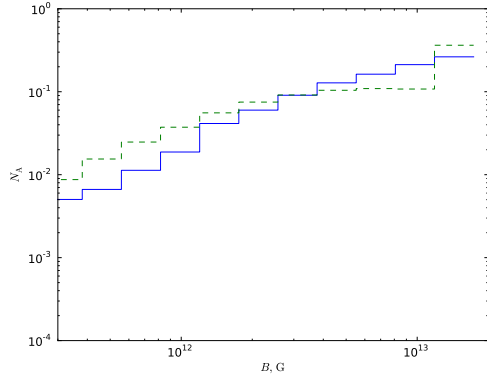
In the solar neighborhood ( $R_{\text{solar}} < 2 \text{ kpc}$  and  $|z| < 0.5 \text{ kpc}$ ) we predict  $\sim 35 - 40\%$  of Accretors and slightly more ( $\sim 40 - 45\%$ ) subsonic Propellers with only  $\sim 18 - 20\%$  of Ejectors. Contributions of others stages are negligible. In total, in the solar proximity ( $R_{\text{solar}} < 2 \text{ kpc}$  and  $|z| < 0.5 \text{ kpc}$ ) there are 0.33% of all NSs. This gives us, for  $N_{\text{NS}} = 10^9$  the number density in the solar neighborhood  $n_0 \approx 3 \times 10^{-4} \text{ pc}^{-3}$ , in good correspondence with recent results by Ofek (2009) and with earlier studies.

## 4 DISCUSSION

Available estimates of the number of Accretors should be taken with care, since there are several effects which act in inhibiting accretion, as it is discussed below. I.e., the number of *observable* Accretors can be much smaller. Still, low velocity INs moving through a high density medium (the only ones with high enough luminosities to be potentially detected) and with strong magnetic fields can become accretors after  $\lesssim$  few Gyr. It was shown above that INs like the M7, which have magnetic fields higher than those typical of radio pulsars, and at the same time have not very large spatial velocities, are the most favored as Accretors predecessors.

Although the number of AINSs might be even larger than that originally estimated in Paper I, the conclusion that at low fluxes Accretors outnumber cooling isolated NS (Coolers) is based on the assumption that the luminosity corre-





**Figure 3.** The dashed line represents the field distribution after the immediate field decay discussed in sec. 2.1.1.2, i.e. the *decayed* model. The solid line shows the contribution to the number of Accretors for different initial fields in the decayed model, normalized to unity. The velocity distribution is calculated for  $w_1 = 0.4$ . Here  $B$  is the magnetic field value at the magnetic pole. Each bin shows mean fraction of time, among all the stars having such a fields.

sponds to the Bondi accretion rate. This is a quite controversial issue. Blaes et al. (1995) have shown that for typical ISM densities ( $n \approx 1 \text{ cm}^{-3}$ ) accretion rate does not exceed  $\sim \text{few } 10^9 \text{ g s}^{-1}$ , even if the star velocity drops below  $\sim 60 \text{ km s}^{-1}$ . This is due to the ionization of the ISM surrounding the star by the X-ray radiation which, in turn, produces an increase in the sound speed freezing the accretion rate. However, in Paper I it was shown that the velocity distribution of Accretors peaks at  $\sim 50 \text{ km s}^{-1}$ , and for these velocities the effect is small. So, for most Accretors, heating of the ISM can be neglected, especially if they appear in regions of high ISM density. A further issue is the role played by the star magnetic field in the accretion flow dynamics outside the Propeller stage. On the basis of 2D MHD calculations, Toropina et al. (2003) concluded that only a fraction of the initial (Bondi) flow reaches the star surface, and this fraction decreases with growing magnetic field of a NS. Whether 3D instabilities may counteract this effect is still an open question.

Anyway, if several weak sources without measured proper motions and interpreted as INS candidates can be identified in ROSAT, Chandra or/and XMM-Newton archives (see Turner et al. 2010 and references therein), or discovered by

eROSITA, then it is not trivial to distinguish Coolers from Accretors.

If an INS comes to the Accretor stage only after a long subsonic Propeller episode, its spin period is  $\gtrsim 10^4 \text{ s}$ . Such long periods are not unexpected even if the subsonic Propeller stage is neglected. When a NS starts to accrete it continues to spin down, until it reaches a quasi-equilibrium period,  $P_{\text{eq}} \approx 10^6 \text{ s}$  for  $n = 1 \text{ cm}^{-3}$  (Konenkov & Popov 1997; Prokhorov et al. 2002). The ultra-long spin periods of Accretors could be the best discriminator between this type of sources and Coolers, which are expected to have spin periods  $\lesssim$  few seconds (like the M7 and cooling PSRs). However, at low fluxes it would be extremely difficult to discover pulsations in Coolers, so the non-detection of a periodicity is not a strong argument in favour of an AINS.

Opposite to Coolers, Accretors are expected to show both – spin-up and spin-down – as their periods fluctuate around the quasi-equilibrium value. However,  $\dot{P}$  measurements can be impossible for faint sources with very long periods.

The period of accreting INSs can be significantly shorter than  $\approx 10^6 \text{ s}$  in the case of magnetic fields decaying down to small values ( $\sim 10^9 \text{ G}$ ), although some kind of fine tuning is necessary. As discussed above, to reach accretion in a time shorter than the Hubble time an INS should have at least a magnetic field  $\approx 10^{12} \text{ G}$ . So, decay should not be significant during the first  $\sim 1 \text{ Gyr}$  of the evolution, otherwise a NS spends all its life as an Ejector or a Propeller (Colpi et al. 1998; Livio et al. 1998; Popov & Prokhorov 2000). If the field decays during the Accretor (or even subsonic Propeller) phase, an INS can attain a period  $\sim 10^3\text{--}10^4 \text{ s}$ , since  $P_{\text{eq}}$  is smaller for smaller fields.

Accretors, at variance with coolers, are not expected to be steady sources because of changes in the accretion rate, due to inhomogeneities of the ISM, on a time-scale

$$t \approx \frac{R_G}{v} \sim 3 \times 10^8 v_{10}^{-3} \text{ s}. \quad (20)$$

Note that this time scale is shorter for fainter sources.

Spatial distribution of Accretors and new weaker Coolers are expected to be slightly different, as the first represent much older population, and for the first higher ISM density is favorable for detection in contrast with the second. New (i.e., undiscovered, yet) Coolers according to Posselt et al. (2008) are expected to be found

at distances  $\sim 1$  kpc. So, they should be relatively bright,  $\sim 10^{31}$  erg s $^{-1}$ . Accretors cannot be that bright, and so they are expected to be found closer. Young Coolers should trace star-forming regions. Accretors, which already experienced long evolution in the Galactic potential, should be distributed more smoothly. However, for them to be detectable it is important to be inside regions of relatively high ISM density.

The X-ray spectrum of a NS accreting at low rate from the ISM is very similar to those of cooling INSS, at least in the case when the latter has a H atmosphere (Treves et al. 2000 and references therein). Nevertheless, for the same luminosity, the effective temperature of an Accretor is higher and, hence, the spectrum is harder because of significantly reduced emitting area. For typical values of the star mass and radius, the hot polar cap size is

$$R_{\text{cap}} \sim 9.5 \times 10^3 \mu_{30}^{-2/7} v_{10}^{-3/7} n^{1/7} \text{ cm}. \quad (21)$$

This is smaller than the size of a typical emitting area in Coolers. Spectra of Accretors are expected to be harder than those of Coolers.

We summarize some differences between Accretors and cooling NSs (Coolers) in Table 2.

## 5 CONCLUSIONS

After the first of the M7 have been discovered (Walter et al. 1996), several authors proposed and discussed that they can be AINSs (Walter et al. 1996; Konenkov & Popov 1997; Neuhäuser & Trümper 1999). Though, it appeared that it is not so. The M7 are young NSs with relatively large fields. Probably, they are related to evolved magnetars (Popov et al. 2010). Here we demonstrate that in future the M7 and similar sources are expected to become AINS if their magnetic fields do not decay significantly. Even a relatively long stage of subsonic Propeller (Ikhsanov 2001) cannot prevent accretion. This is a good news for observers. Probably, telescopes like eROSITA aboard Spektr-RG will be able to detect AINS, soon. However, the question of the accretion efficiency is still on the list (Toropina et al. 2003).

The distribution over evolutionary stages strongly depends on kick velocity distribution, initial magnetic field distribution and field evolution. Because of that precise predictions are not possible now. This shows how important is to detect old isolated NSs as Accretors (or, less proba-

ble, other stages) to learn more about initial properties and evolution of INSS.

## ACKNOWLEDGMENTS

S.P. thanks Profs. Jose Pons, Aldo Treves, and Roberto Turolla for discussions. This work was supported by the RFBR grants 07-02-00961 and 09-02-00032, and by the Federal program for scientific and educational personnel.

## REFERENCES

- Arzoumanian Z., Chernoff D. F., Cordes J. M., 2002, *ApJ*, 568, 289
- Blaes O., Blandford R., Madau P., Koonin S., 1990, *ApJ*, 363, 612
- Blaes O., Madau P., 1993, *ApJ*, 403, 690
- Blaes O., Rajagopal M., 1991, *ApJ*, 381, 210
- Blaes O., Warren O., Madau P., 1995, *ApJ*, 454, 370
- Colpi M., Turolla R., Zane S., Treves A., 1998, *ApJ*, 501, 252
- Davies R. E., Pringle J. E., 1981, *MNRAS*, 196, 209
- Faucher-Giguère C., Kaspi V. M., 2006, *ApJ*, 643, 332
- Gotthelf E. V., Halpern J. P., 2009, *ApJL*, 695, L35
- Haberl F., 2007, *Astrophys. & Space Sciences*, 308, 181
- Halpern J. P., Gotthelf E. V., Camilo F., Seward F. D., 2007, *ApJ*, 665, 1304
- Hindmarsh A. C., 1983, *IMACS Transactions on Scientific Computation*, 1, 55
- Hobbs G., Lorimer D. R., Lyne A. G., Kramer M., 2005, *MNRAS*, 360, 974
- Ikhsanov N. R., 2001, *A&A*, 368, L5
- Kaplan D. L., 2008, in C. Bassa, Z. Wang, A. Cumming, & V. M. Kaspi ed., 40 Years of Pulsars: Millisecond Pulsars, Magnetars and More Vol. 983 of American Institute of Physics Conference Series, Nearby, Thermally Emitting Neutron Stars. pp 331–339
- Keane E. F., Kramer M., 2008, *MNRAS*, 391, 2009
- Konenkov D. Y., Popov S. B., 1997, *Astronomy Letters*, 23, 498
- Lipunov V. M., 1992, *Astrophysics of Neutron Stars*
- Livio M., Xu C., Frank J., 1998, *ApJ*, 492, 298

**Table 2.** Comparison of properties of dim Accretors and Coolers

	Accretors	Coolers
Spectrum	Harder, $\sim$ hundreds eV	Softer, $\sim$ 100 eV
Spin periods	Very long, $> 10^5$ s	Shorter, $\sim$ 10 s
$\dot{p}$	Variable	Stable spin-down
Distance	Close, $\sim 100 - 200$ pc	Further away, $\sim 1$ kpc
Luminosity	Low, $\sim 10^{29}$ erg s $^{-1}$	Higher, $\sim 10^{31}$ erg s $^{-1}$
Variability	Variable, $\Delta t \sim$ weeks – yrs	Stable
Spatial distribution	Towards higher gas density	Towards starforming regions

- Lyne A. G., Lorimer D. R., 1994, *Nature* , 369, 127
- Madau P., Blaes O., 1994, *ApJ* , 423, 748
- Manning R. A., Jeffries R. D., Willmore A. P., 1996, *MNRAS* , 278, 577
- McLaughlin M. A., Lyne A. G., Lorimer D. R., Kramer M., Faulkner A. J., Manchester R. N., Cordes J. M., Camilo F., Possenti A., Stairs I. H., Hobbs G., D’Amico N., Burgay M., O’Brien J. T., 2006, *Nature* , 439, 817
- Mereghetti S., 2008, *A&ARv* , 15, 225
- Miyamoto M., Nagai R., 1975, *PASJ*, 27, 533
- Neuhäuser R., Trümper J. E., 1999, *A&A* , 343, 151
- Ofek E. O., 2009, *PASP* , 121, 814
- Ostriker J. P., Rees M. J., Silk J., 1970, *ApLett* , 6, 179
- Paczynski B., 1990, *ApJ* , 348, 485
- Pons J. A., Miralles J. A., Geppert U., 2009, *A&A* , 496, 207
- Popov S. B., 2008, *Physics of Particles and Nuclei*, 39, 1136
- Popov S. B., Colpi M., Treves A., Turolla R., Lipunov V. M., Prokhorov M. E., 2000, *ApJ* , 530, 896
- Popov S. B., Pons J. A., Miralles J. A., Boldin P. A., Posselt B., 2010, *MNRAS* , 401, 2675
- Popov S. B., Prokhorov M. E., 2000, *A&A* , 357, 164
- Popov S. B., Prokhorov M. E., 2007, *Physics Uspekhi*, 50, 1123
- Popov S. B., Turolla R., Possenti A., 2006, *MNRAS* , 369, L23
- Posselt B., Popov S. B., Haberl F., Trümper J., Turolla R., Neuhäuser R., 2008, *A&A* , 482, 617
- Prokhorov M. E., Popov S. B., Khoperskov A. V., 2002, *A&A* , 381, 1000
- Shakura N. I., 1975, *Soviet Astronomy Letters*, 1, 223
- Shvartsman V. G., 1971, *Soviet Astronomy*, 14, 662
- Toropina O. D., Romanova M. M., Toropin Y. M., Lovelace R. V. E., 2003, *ApJ* , 593, 472
- Treves A., Colpi M., 1991, *A&A* , 241, 107
- Treves A., Turolla R., Zane S., Colpi M., 2000, *PASP* , 112, 297
- Turner M. L., Rutledge R. E., Letcavage R., Shevchuk A. S. H., Fox D. B., 2010, *ArXiv e-prints*
- Walter F. M., Wolk S. J., Neuhäuser R., 1996, *Nature* , 379, 233
- Zane S., Turolla R., Zampieri L., Colpi M., Treves A., 1995, *ApJ* , 451, 739